

KORT, TRÉ OG MYNSTUR

Stærðfræði á Íslandi 2011
Reykholt, Borgarfirði

Sergey Kitaev



Pavel Salimov



- Byggt á grein sem er á lokasprettinum með

Christopher Severs



- Greinin fjallar um

- lagnet

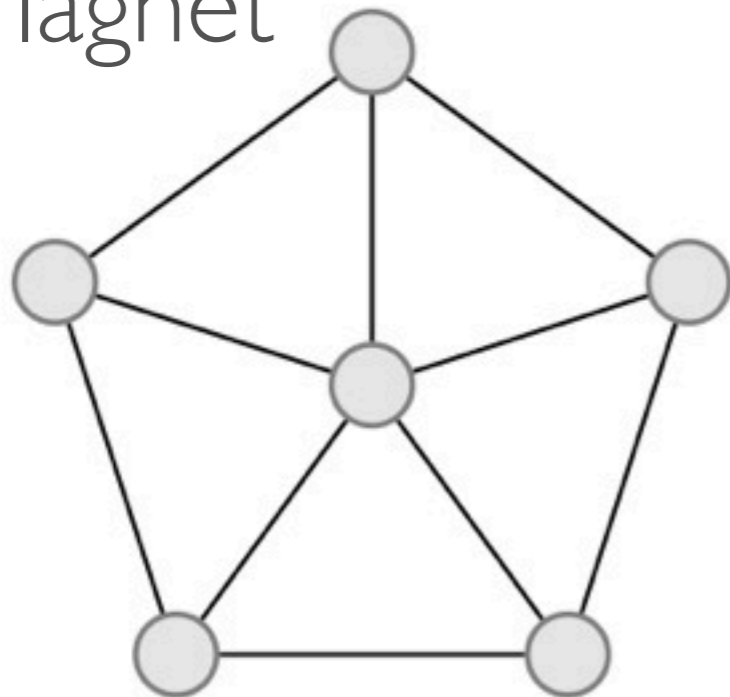
- tré

- umraðanir

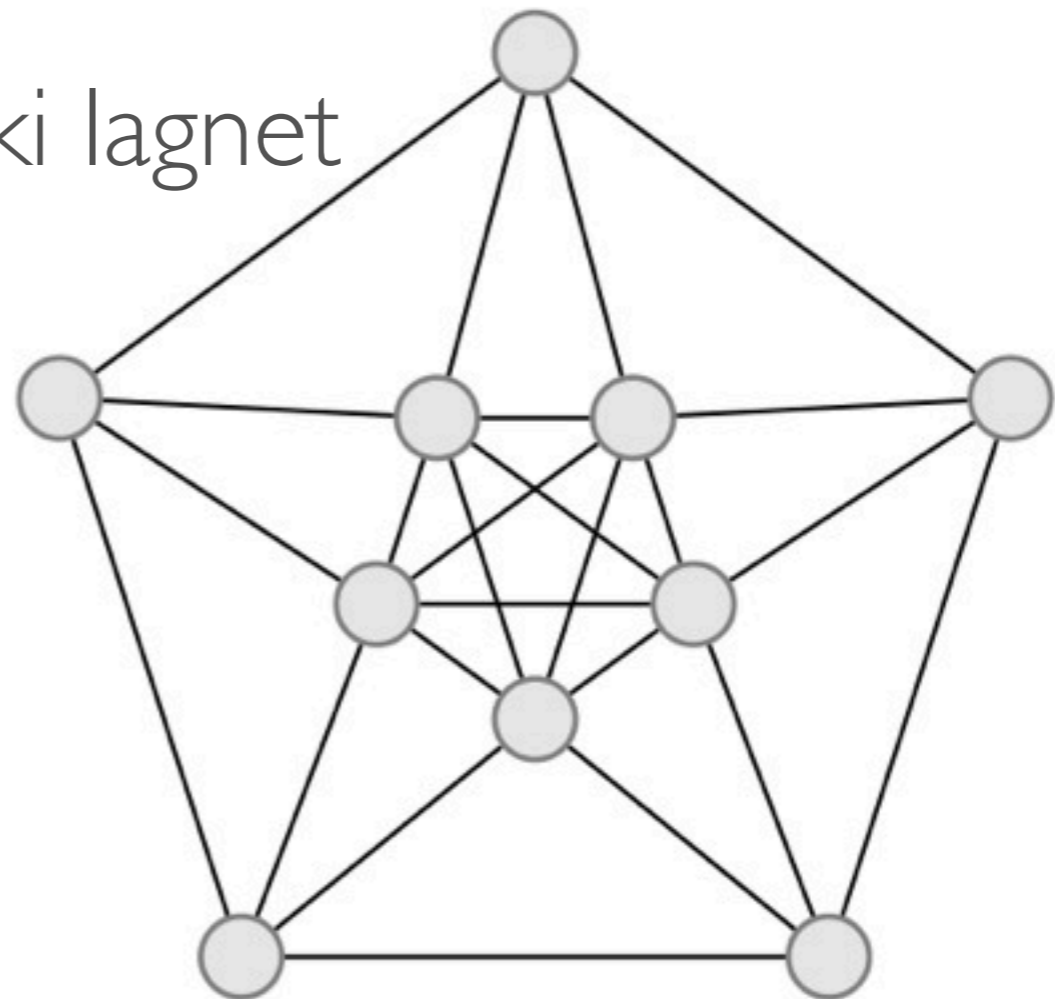
LAGNET

- *Lagnet* er hægt að teikna í sléttuna án þess að leggirnir skerist

lagnet

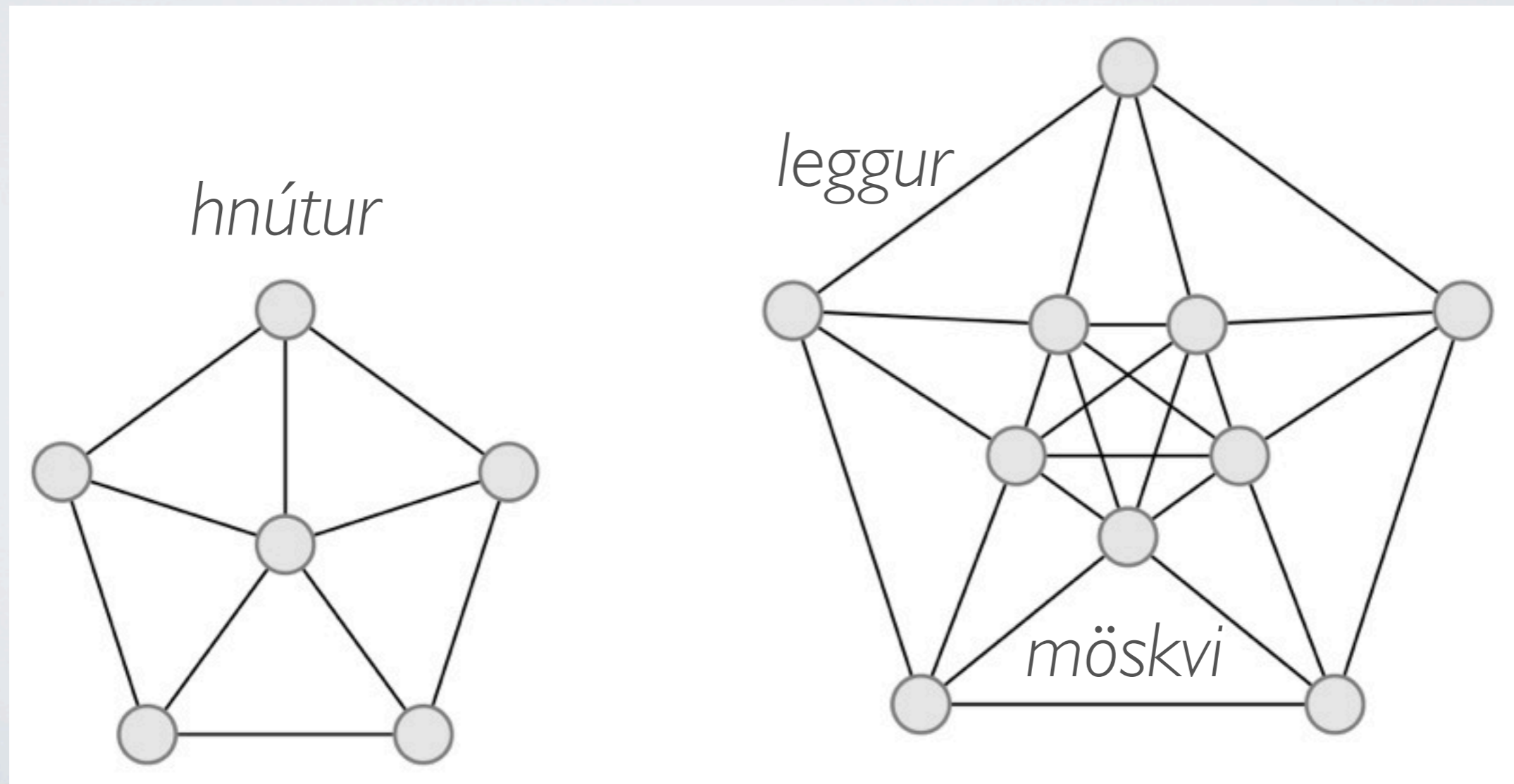


ekki lagnet



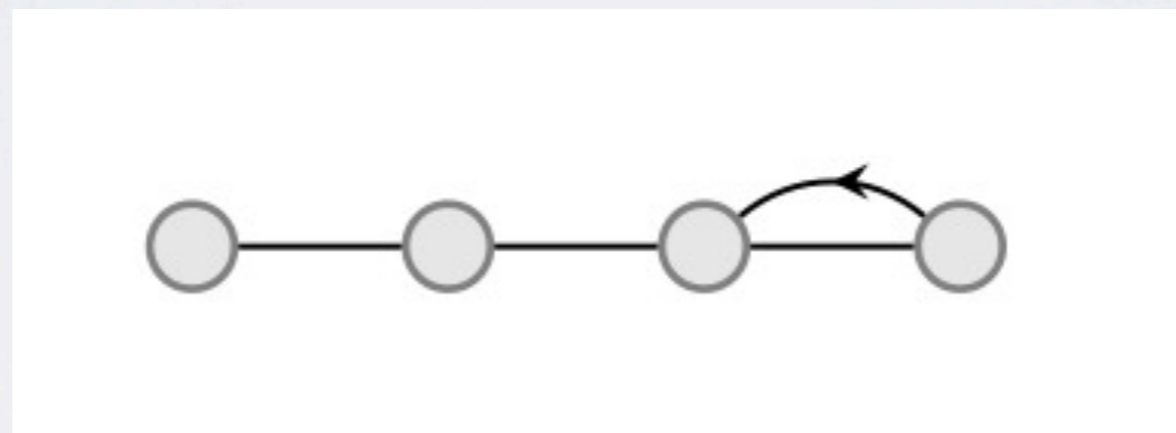
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RÓTUÐ OG ÓAÐSKILJANLEG

- Lagnet er *rótað* með því að velja legg og átta hann

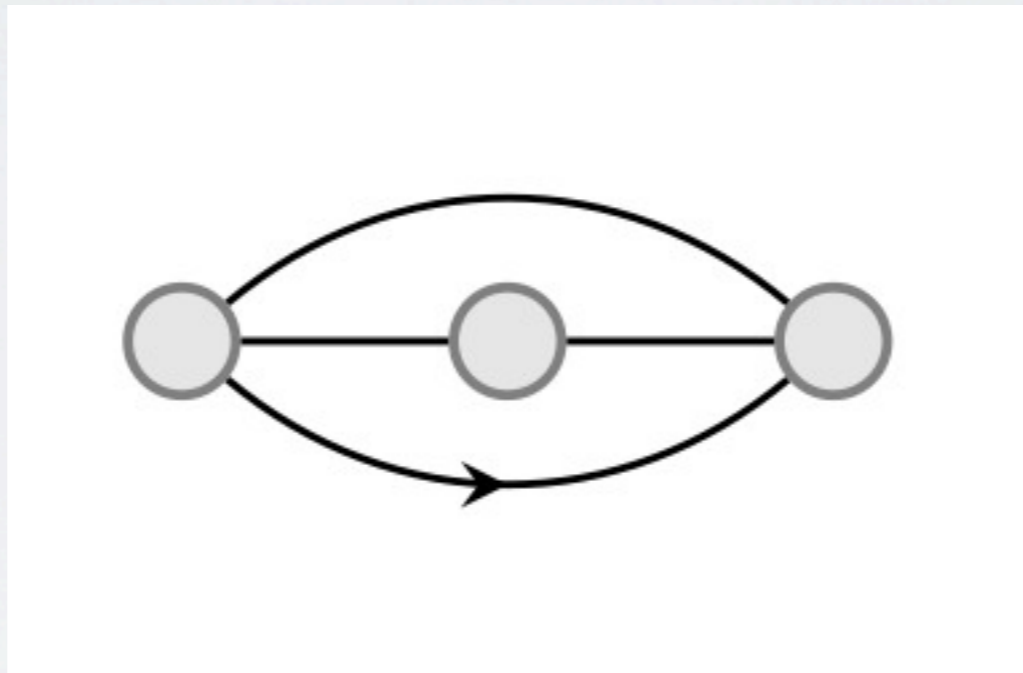


aðskiljanlegt

- Lagnet er *aðskiljanlegt* ef það dettur í sundur þegar við fjarlægjum hnút

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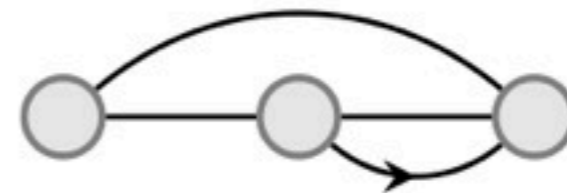
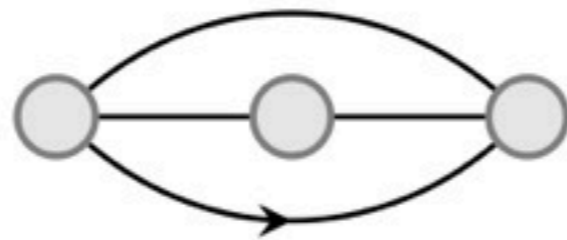
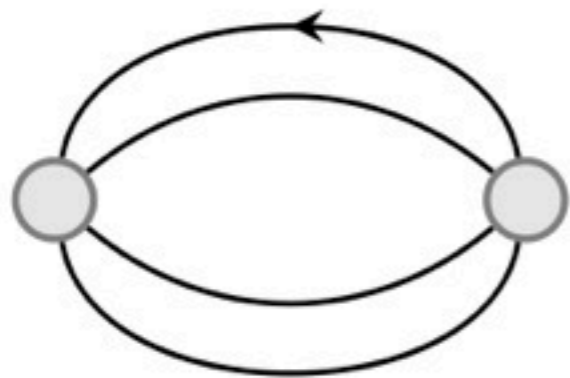


óaðskiljanlegt

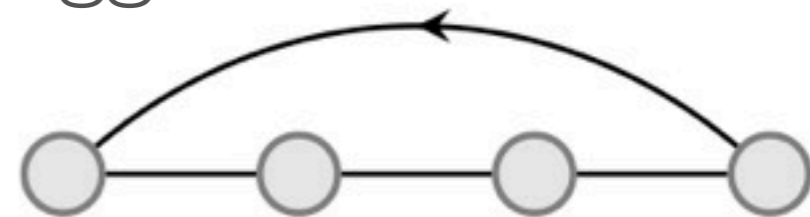
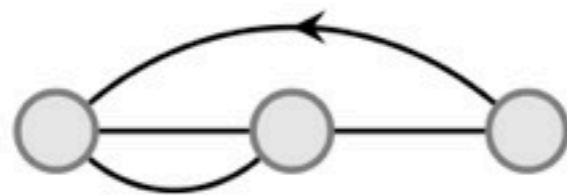
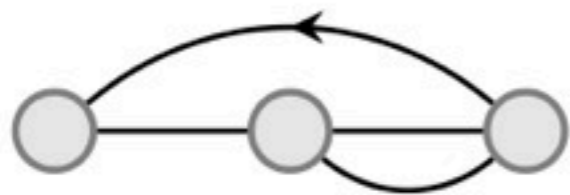
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KORT

- Óaðskiljanleg rótuð lagnet verða hér eftir kölluð *kort*



Öll kort með 4 leggi





TALNING

1917-2002

- Tutte sannaði að fjöldi korta með $n+1$ legg er

$$\frac{4(3n)!}{n!(2n+2)!}$$

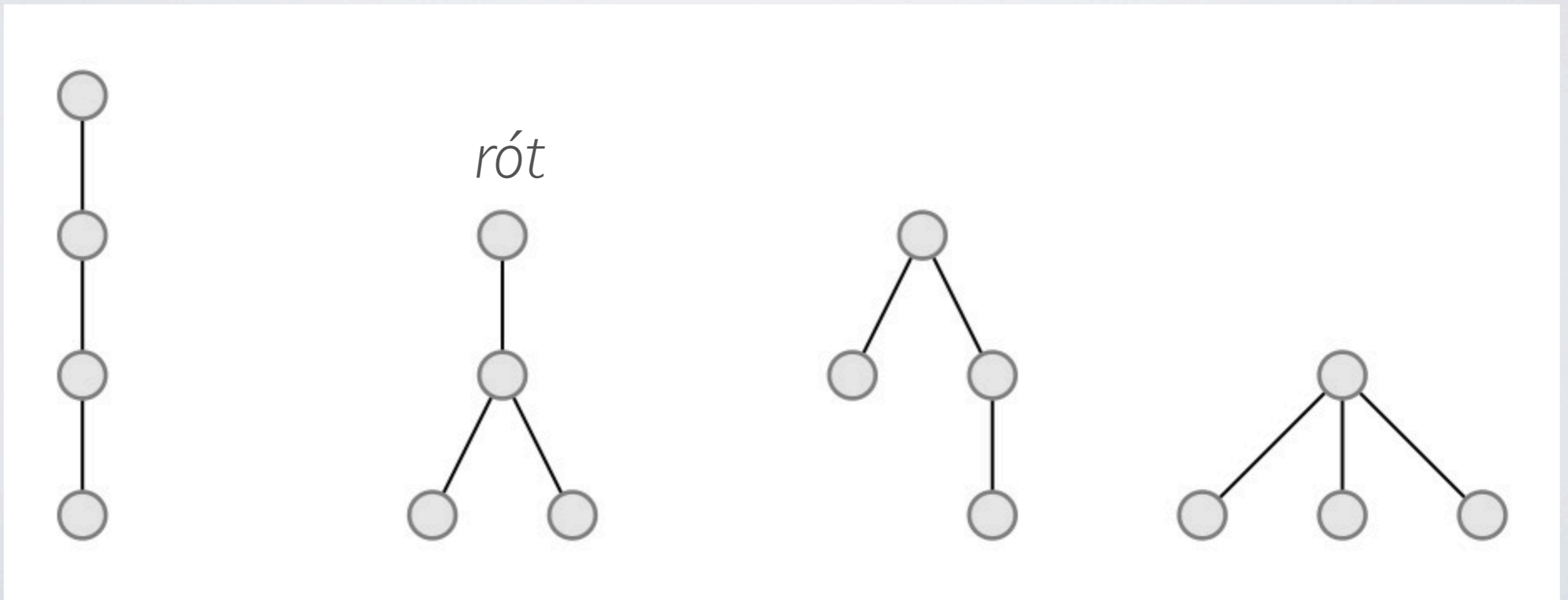
- Fyrstu tölurnar eru 1, 2, 6, 24, 110, 546
- Þetta eru runa [A046646](#) í OEIS

AF HVERJU KORT?

- Jafnmörg og umraðanir sem eru raðanlegar með tveimur umferðum í gegnum stafla (Goulden & West 1996)
- Koma fyrir í líkani fyrir tvívíð skammtafræðileg þyngdarsvið (Schaeffer & Zinn-Justin 2004)
- Tengjast fleiri fléttufræðilegum fyrirbærum

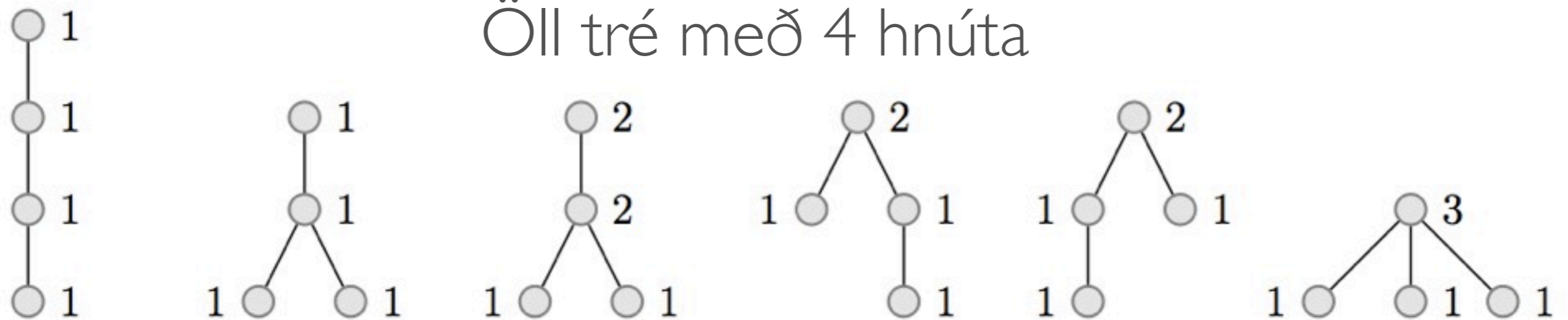
TRÉ

- Net með engar rásir kallast *tré*



BETA(1,0)-TRÉ

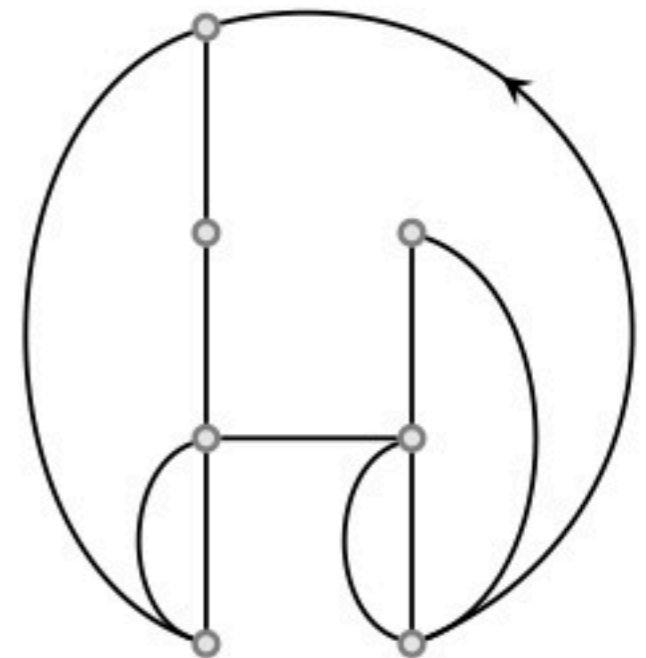
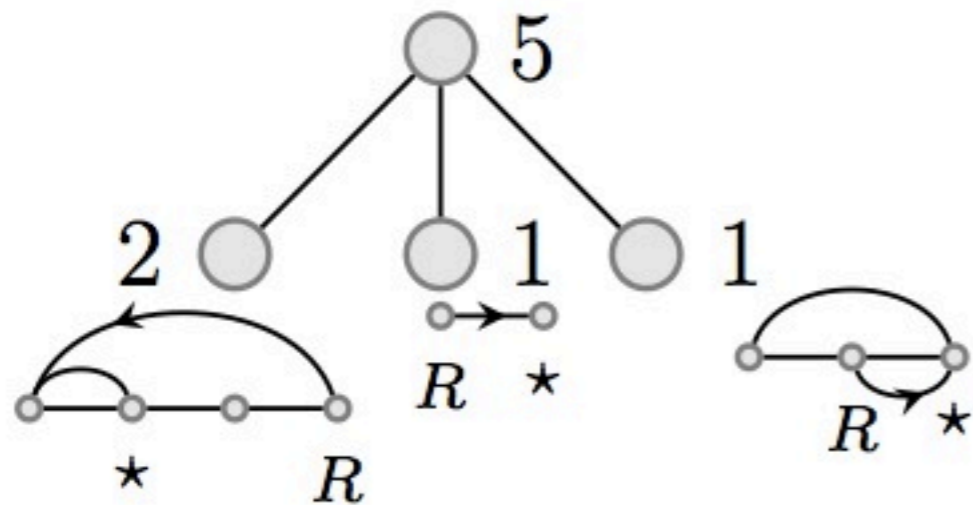
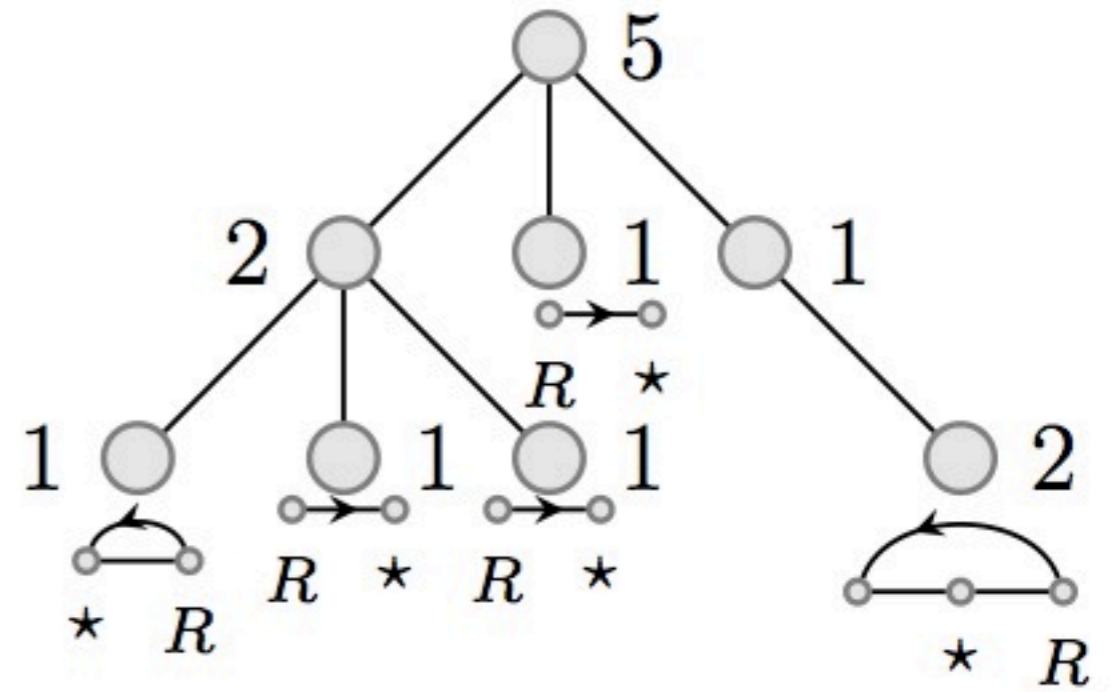
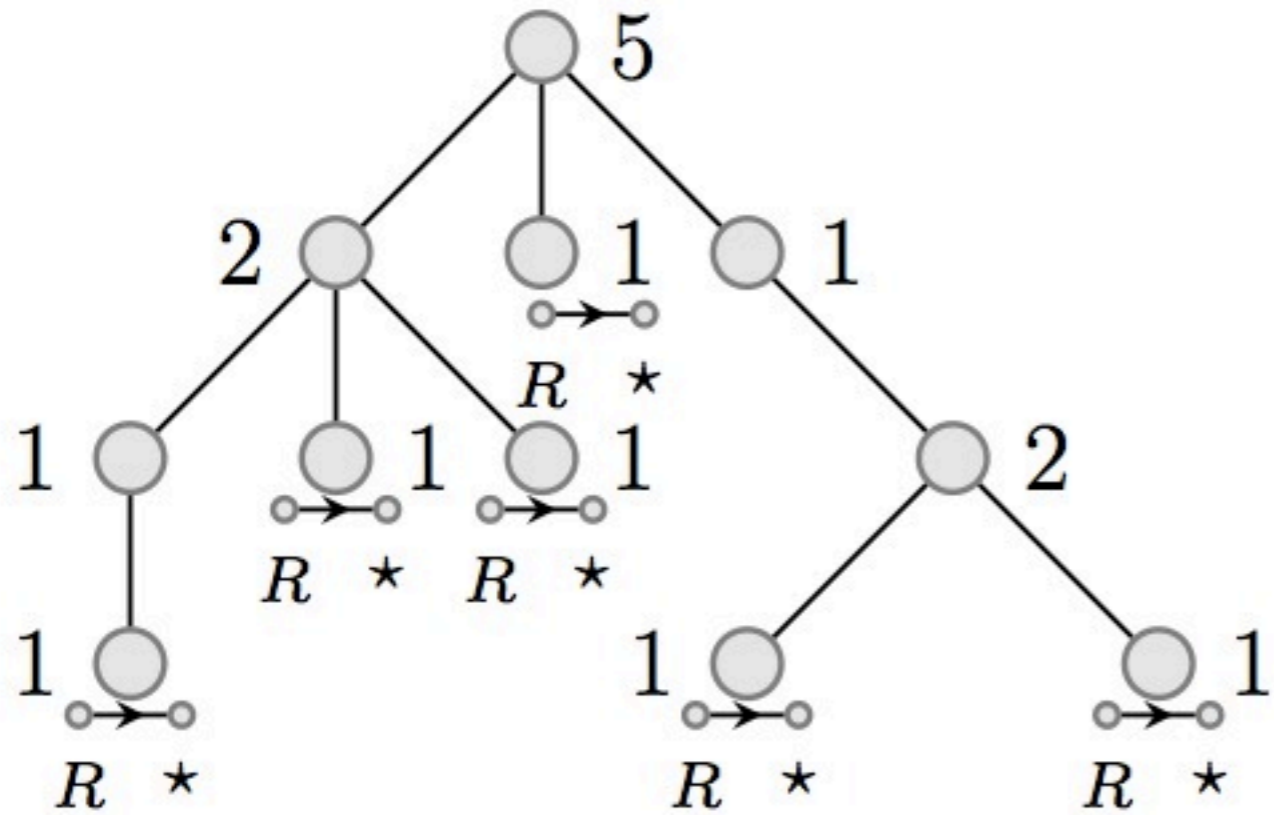
- Cori, Jacquard og Schaeffer (1997) skilgreindu beta(1,0)-tré
- Teiknuð í sléttuna, merkt eftir ákveðnum reglum



TENGSLVIÐ KORTIN

- Þessi tré voru sérstaklega hönnuð til þess að rannsaka kortin
- Gagntæk samsvörun á milli trjáanna og kortanna

FRÁ TRÉ TIL KORTS



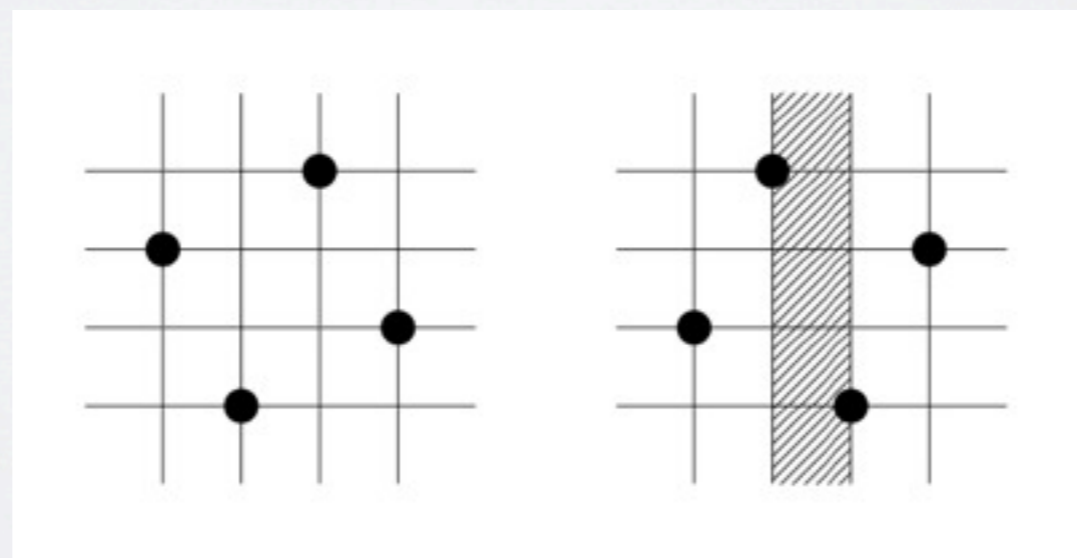
UMRÆÐANIR

- Umröðun af lengd n er listi þar sem tölurnar 1 til n koma nákvæmlega einu sinni fyrir
- 5642173 er umröðun af lengd 7

TENGLSLVIÐ TRÉN



- Claesson, Kitaev og Steingrímsson (2009) bjuggu til gagntæka vörpun milli trjáanna og umraðana sem forðast mynstrin

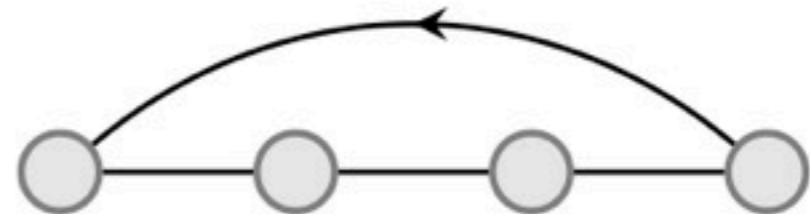
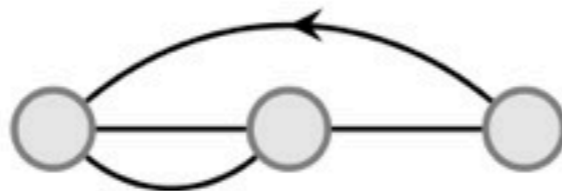
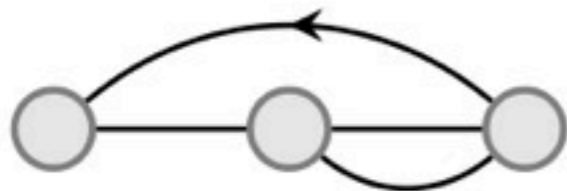
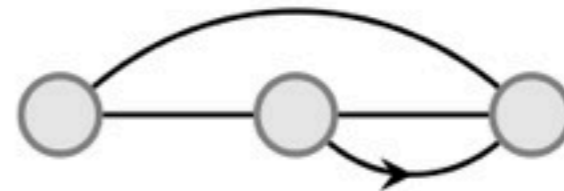
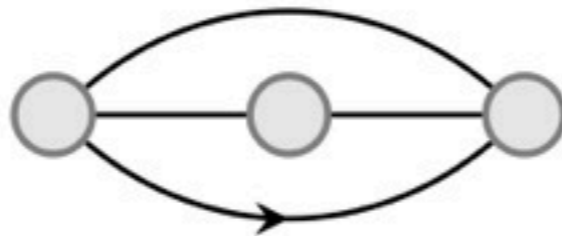
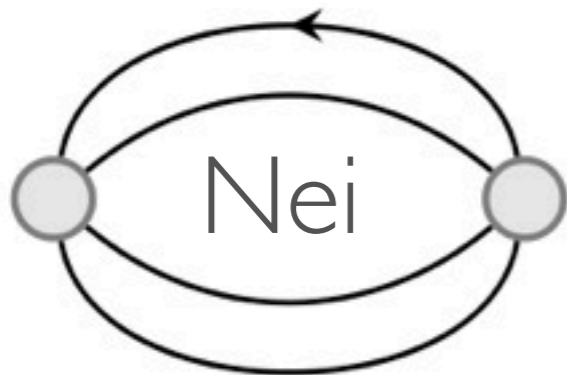


EIGINLEIKAR

- Hvernig flytjast eiginleikar á milli
 - korta
 - trjáa
 - umraðana

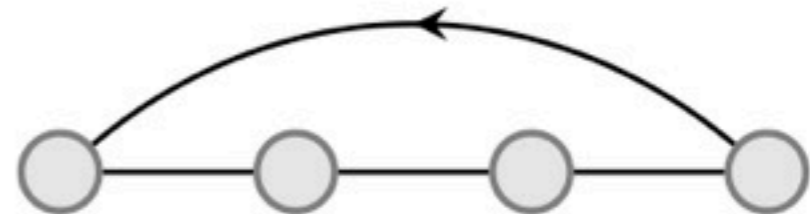
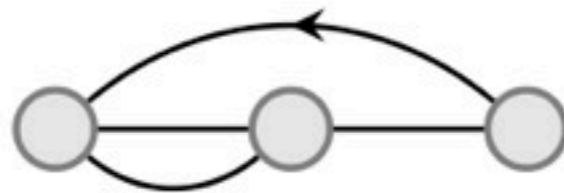
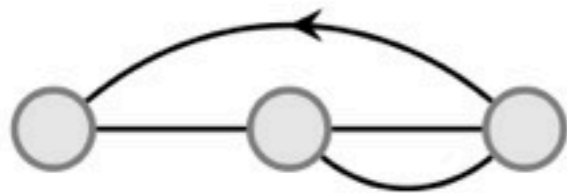
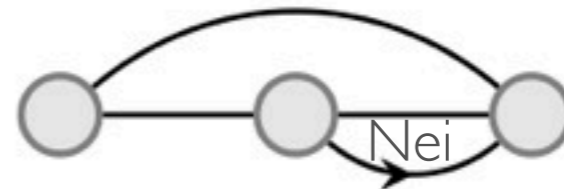
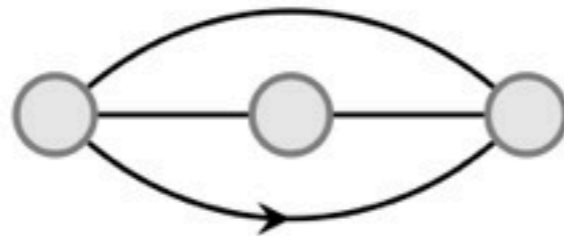
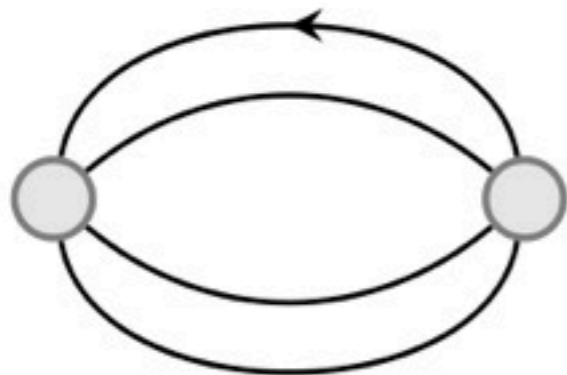
FRUMSTÆÐ KORT

- Kort er *frumstætt* ef það hefur ekki innanverðan möskva með tvo leggi



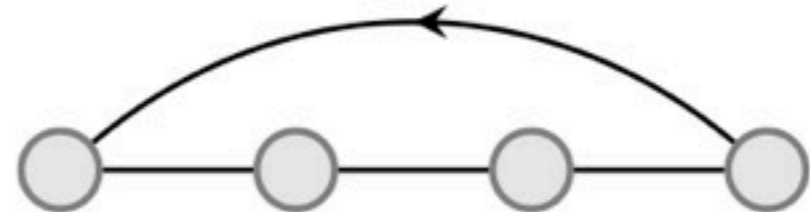
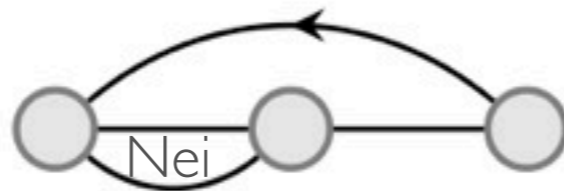
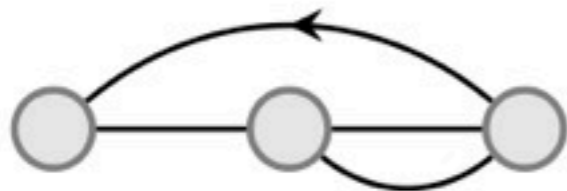
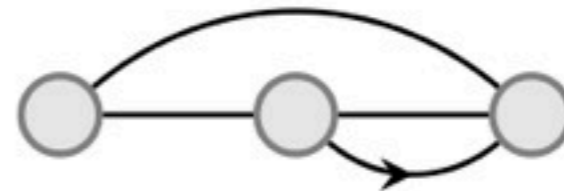
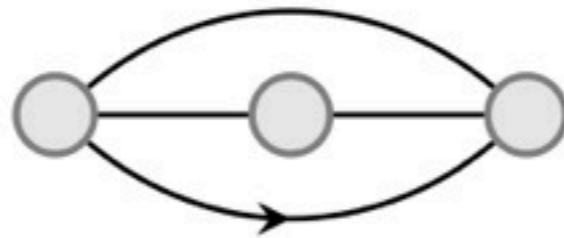
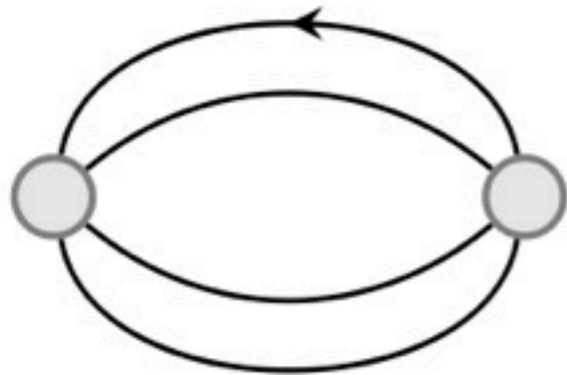
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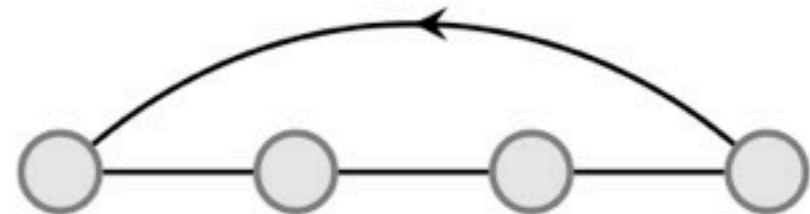
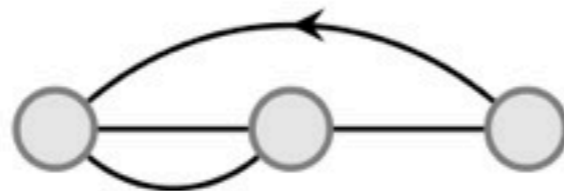
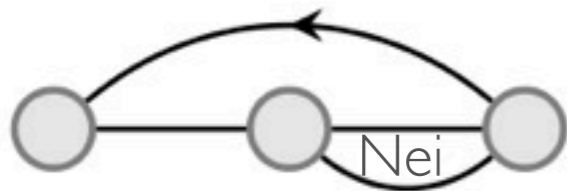
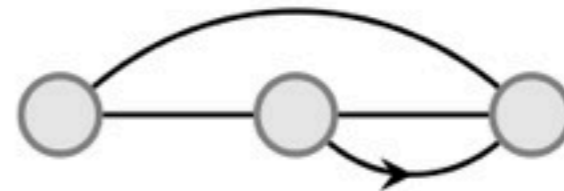
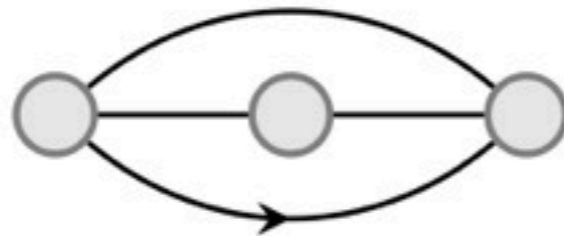
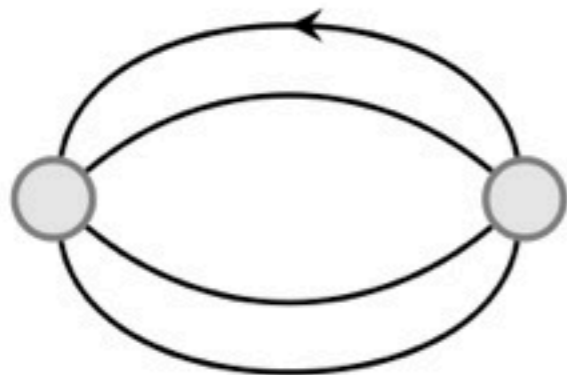
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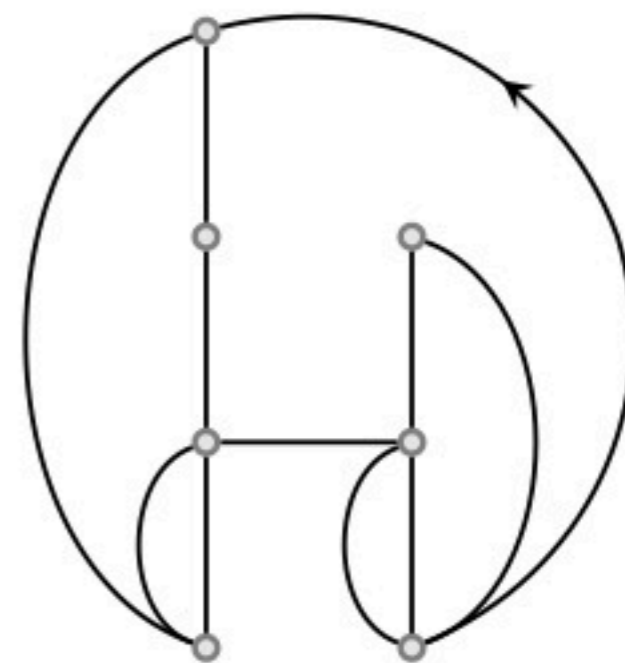
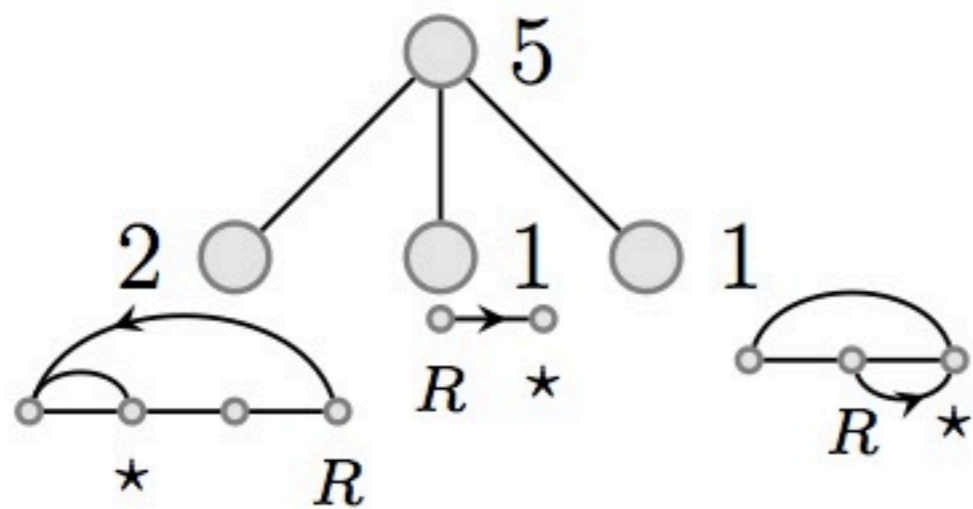
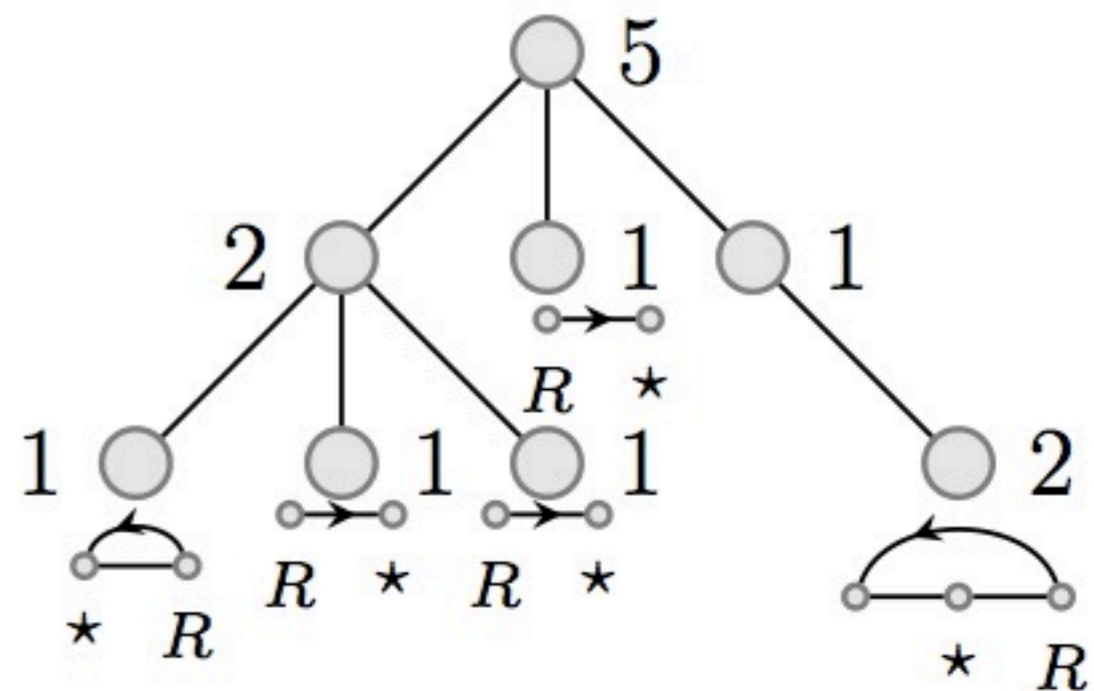
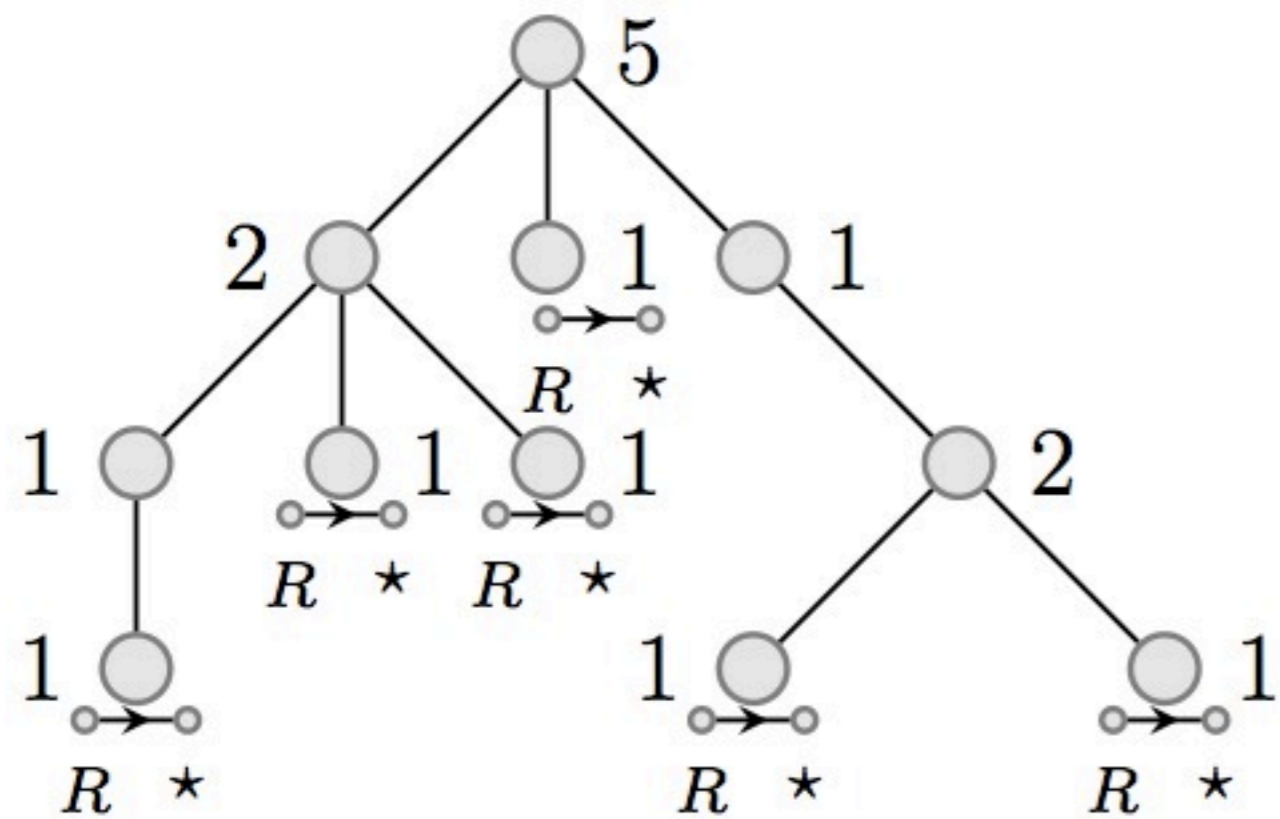
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FRUMSTÆÐ TRÉ

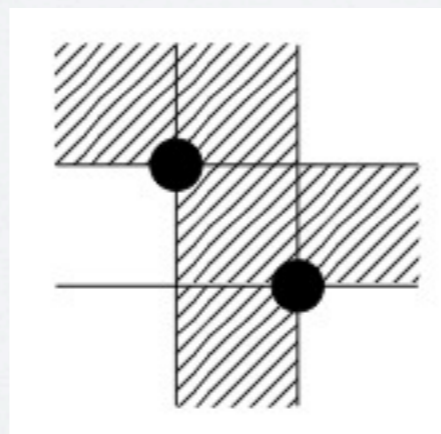
- Tré er *frumstætt* ef það samsvara frumstæðu korti
- Setning (Kitaev, Salimov, Severs, Ú). Tré er frumstætt þþaa það hafi ekki hnút sem er einbirni og hefur hæsta mögulega gildi



- Tveir “slæmir” hnútar gefa tvo “slæma” möskva

FRUMSTÆÐ UMRAÐANIR

- Umröðun er *frumstæð* ef hún samsvarar frumstæðu korti
- Setning (Kitaev, Salimov, Severs, Ú). Umröðun er frumstæð þá hún forðast möskvamynstrið



VANGAVELTUR

- Í greininni skoðum við einnig kort sem hafa ekki möskva með k leggjum, $k = 3$ og 4
- Gaman væri að skoða hvernig eiginleikar trjáa og umraðana flytjast yfir á kortin

Spurningar?